

# Computing the Expansion History of the Universe

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7 December 2010

In the complete expanding, relativistic model of our universe, the metric (time and space) depends on the energy density (mass,+ kinetic+ potential per volume) and vice versa. The scale factor that describes the expansion,  $a(t)$ , leads directly to observable quantities like the age of the universe, the redshift of a galaxy, the galaxy's luminosity distance, distance modulus, and angular diameter distance. In the first part of this project, we will use  $a(t)$  to compute these observable quantities. In the second part, we will use the general solution to the Friedman Equation to determine  $a(t)$  for any universe of your choosing. In part 3, you'll explore the parameter space to determine how close Ryden's Benchmark model comes to the current best-measured parameters. You'll also compare philosophically satisfying cosmologies to the measured cosmology.

## Part 1 – Observable Quantities

If a distant galaxy emits light of a wavelength,  $\lambda_e$ , ( $e$  is for emitted) we will observe it redshifted from its original,  $\lambda_o$ , ( $o$  is for observed). The redshift is defined as  $z = (\lambda_o - \lambda_e)/\lambda_e$ . The redshift is also related to the scale factor by: (Ryden Eqn 3.46)

$$z(t) = \left(\frac{1}{a(t)}\right) - 1 \quad 1$$

Spectral lines of hydrogen are pretty simple to measure. For example, the Balmer lines of hydrogen are routinely measured in undergraduate labs. In galaxies with strong hydrogen lines, we can determine the observed wavelength. Since we already know the emitted wavelengths from our laboratory studies, we can find the redshift pretty easily. This in turn lets us measure the scale factor at the time when the light was emitted.

The proper distance is another interesting quantity to compute. (Ryden Eqns. 3.28 and 3.38)

$$D_p = c \int \frac{dt}{a(t)} \quad 2$$

where  $c$  is the speed of light. This is the line-of-sight distance to the source at the time we observe it. In an expanding universe the distance was smaller at the time the light was emitted. Now here's the interesting thing: we can't simply go and measure it with a ruler or some such device. Light is our best measuring device and it can't be both there and here at the same time. The things we can measure are the

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brightness of a distant source and its angular size on the sky. For these, we need the transverse proper distance. It is just the line-of-sight proper distance with the usual additional curvature terms.

$$D_{pt} = \begin{cases} (R_0/\kappa)\sin(D_p\kappa/R_0) & \text{for } (1 - \Omega_0) < 0, \kappa = +1 \\ D_p & \text{for } (1 - \Omega_0) = 0, \kappa = 0 \\ (-R_0/\kappa)\sin(-D_p\kappa/R_0) & \text{for } (1 - \Omega_0) > 0, \kappa = -1 \end{cases} \quad 3$$

where  $R_0$  is the radius of the curvature of space and  $\kappa$  determines if the curvature is positive, flat, or negative. It relates directly to the curvature term in the Friedman equation:

$$-\frac{\kappa}{R_0^2} = \left(\frac{H_0^2}{c^2}\right) |1 - \Omega_0| \quad (\text{Ryden 4.31}).$$

The measured brightness of a source depends on how far away it is: measured flux = true luminosity/ $(4\pi D_L^2)$  where the luminosity distance,  $D_L$ , is related to the transverse proper distance by

$$D_L = D_{pt}(1 + z) \quad 4$$

Similarly, the observed width of a galaxy on the sky,  $\Delta\theta$ , is related to the galaxy's diameter: true galaxy diameter =  $D_A \Delta\theta$  where  $D_A$  is the angular diameter distance. The angular diameter distance is related to the transverse proper distance by

$$D_L = D_{pt}/(1 + z) \quad 5$$

These are derived in Ryden Ch. 7 for the flat universe where  $D_p = D_{pt}$ .

Finally, it is rare to measure a luminosity or flux directly. Usually we find ourselves integrating the flux and describing the brightness of a source by its magnitude. The absolute magnitude,  $M$ , of a source is given by (Ryden Eqn. 7.48)

$$M = m - 5 \log\left(\frac{D_L}{10pc}\right) \quad 6$$

where  $m$  is the observed magnitude. The 2<sup>nd</sup> term is called the distance modulus and we will calculate it here,  $DM = 5 \log(D_L/10pc)$ . In general we don't want to know what we measure, but how bright the galaxy is in relation to other galaxies. A bright galaxy will look dim if it's far away. The distance modulus rescales the measured brightness to how bright it would be if it was located at a respectable distance of 10 pc. When comparing the properties of galaxies, we often want to know the brightness as if all the galaxies were equidistant.

All of these measurable quantities can be computed if we can just figure out  $a(t)$ . There are a few cases where the scale factor can be computed uniquely and in this part of the project, its good to start with one of those. In particular, we note that the Einstein-deSitter universe, which is composed only of matter, is easily computed.

$$H_0(t - t_0) = \frac{2}{3}(a^{\frac{3}{2}} - 1) \quad 7$$

where  $H_0$  is the Hubble constant. (note:  $H_0$  is not a function of  $t-t_0$ , but rather, the

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left-hand side is  $H_0 \times (t - t_0)$ .) The function is plotted as a dotted line in Ryden's Figure 6.1. We will choose  $t_0 = \text{now}$ , and measure  $t$ , as a time in the past or future. When  $t = t_0$ , the left-hand side is zero. What is  $a(\text{now})$ ? In summary, we can use  $a(t)$  to understand redshift, and the distance factors.

**General Instructions:** If you wish to do this assignment without the step-by-step instructions, feel free to pick any computing language of your choice. Make an array of  $\log(a)$  from -6 to +3, incrementing in steps of 0.1 or so. Make additional arrays from the first to hold the values of  $a$ ,  $z$ ,  $H_0(t-t_0)$ . Define the constants  $H_0$  and  $c$ . Be careful with the units. Debug the results using columns A-F in the spreadsheet shown in Figure 3. What is the age of the universe? Next, integrate Eqn. 2 being very careful to set the integration limits from  $t_0$  to  $t$ . (See Eqn. 8 below) Set  $D_p = 0$  at  $t = t_0$  and then integrate backward to  $t$  in the past and forward to  $t$  in the future. Again, check that  $D_p$  is right using the spreadsheet in Figure 3. Finally, compute  $D_{pt}/D_H$ ,  $D_A/D_H$ ,  $D_L/D_H$ , and  $DM$ . Recreate the plots in Figures 1 and 4. Skip ahead to page 5 where it says **Report**.

**Excel Instructions:** If you prefer more explanation and a detailed approach, here's how to do the computation in Excel.

A) In later calculations, it won't be possible to solve for  $a$  as a function of  $H_0(t - t_0)$  so we want to get good at doing this without having to invert Eqn. 7. We'll also want to explore phenomena at lots of different times, so we'll start with  $\log(a)$  instead of  $a$ . Take a look at the example spreadsheet below to see how it's done. For starters, create a spreadsheet column  $\log(a)$ . Compute  $a$  from the  $\log(a)$  and  $H_0(t - t_0)$  from  $a$ . Recreate the dotted line in Ryden's Figure 6.1 for yourself.

B) We will later be computing a wide range of times & redshifts. We'll want to explore  $10^{-6} < a < 10^4$ . This will be easier if we start with  $\log(a) = -6$  and go to  $\log(a) = +3$  in steps of 0.01. It's a good idea to change the first column now. Doing this later may introduce bugs and become very frustrating. Start the  $\log(a)$  column at -6 and increment it up to +3 in steps of 0.01. See the spreadsheet in Figure 3. Move the data on the plot so that you just plot the part you want.

Now, let's compute the observables. See the spreadsheet below.

C) Emission time,  $t-t_0$  is easily computed if we use  $H_0 = 70$  km/s/Mpc. (See column D

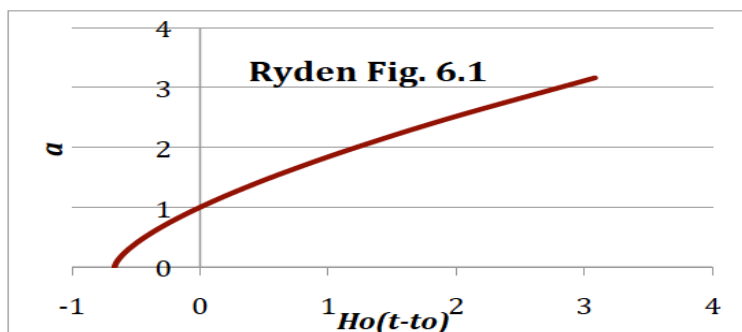


Figure 1: Recreation of the dotted line in the lower panel of Ryden Fig. 6.1 for the Einstein-DeSitter universe. The spreadsheet is shown on the right.

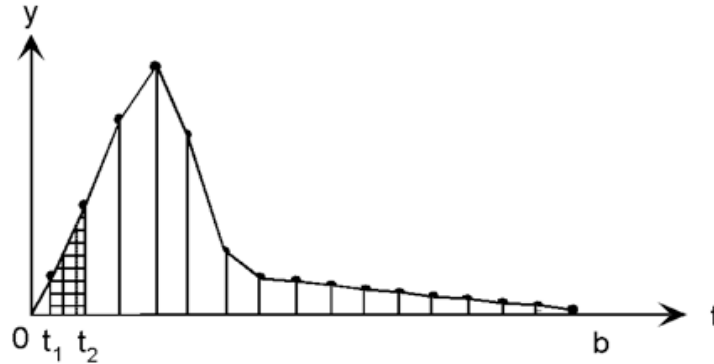
	A	B	C
1	Einstein-DeSitter		
2			
3			
4			$f(a) = 2/3(a^{3/2} - 1)$
5	log(a)	a	$H_0(t-t_0)$
56	-1	0.1	-0.645584816
57	-0.9	0.1258925	-0.636887761
58	-0.8	0.1584893	-0.624602844
59	-0.7	0.1995262	-0.607249937
60	-0.6	0.2511886	-0.582738306
61	-0.5	0.3162278	-0.548114706
62	-0.4	0.3981072	-0.499207571
63	-0.3	0.5011872	-0.430124407
64	-0.2	0.6309573	-0.332541844
65	-0.1	0.7943282	-0.19470281
66	0	1	0
67	0.1	1.2589254	0.27502503
68	0.2	1.5848932	0.66350821
69	0.3	1.9952623	1.21225288
70	0.4	2.5118864	1.987381137
71	0.5	3.1622777	3.082275501
72	0.6	3.9810717	4.628854898
73	0.7	5.0118723	6.813456362
74	0.8	6.3095734	9.89928795
75	0.9	7.9432823	14.25814092
76	1	10	20.4151844

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in Figure 3 to check your work.) Use this to compute the age of the Einstein-deSitter universe (column E). Notice that the emission time is measured backward from now and that age is measured forward from the Big Bang.

D) Compute the redshift column as well as the Hubble Time,  $t_H = 1/H_0$ , and the Hubble Distance,  $D_H = c/H_0$ .

E) The distance factors involve the integral shown in Eqn. 2. To compute this numerically, we'll find the area under the function,  $f(t) = c/a(t)$  where  $f(t)$  is plotted on the y-axis. A schematic of this is shown below.



The area of the shaded trapezoid above is

$$Area = (t_2 - t_1) \left[ \frac{f(t_1) + f(t_2)}{2} \right]$$

Figure 2: Schematic used to describe numerical integration. Note that the y-axis of the plot is  $f(t)$ .

Column G in the spreadsheet below is the area of each trapezoid. The time difference comes from Column D and the scale factors come from Column B. Compute column G.

To integrate Eqn. 2, we need to add up the trapezoidal areas in column G between our integration limits. This is where it gets tricky because we don't want to start at the beginning of the universe, but rather at the current time,  $t_0$ .

$$D_p = c \int_{t_0}^t \frac{dt}{a(t)} \quad 8$$

We want  $D_p$  to be zero at the current time: **put a zero in the column H cell where  $t-t_0 = 0$** . This means that light emitted right now is at zero distance from us. Now we'll add the trapezoids above to find the distance to light emitted in the past. Its simplest if you have an equation like **H9 = H10+G9** in your spreadsheet. For light emitted in the future, add the trapezoids below to find what their distance will be someday after time passes. Make sure column H matches the example below.

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F) We will need to use some IF() statements in Excel to compute the transverse proper distance,  $D_{pt}$ , column. These work by assigning the cell to either the first or second value based on whether the logical test is true or false: **cell value=IF(logical\_test, value\_if\_true, value\_if\_false)**

To set  $D_{pt}$  based on the curvature parameter, we'll need logic like:

$$D_{pt} = \text{IF}((1-\Omega_0) = 0, D_p, \text{IF}((1-\Omega_0) < 1, \text{top Eqn 3, bottom Eqn 3}))$$

This nested IF statement will choose the right equation based on the value of  $(1-\Omega_0)$ .

In the spreadsheet below, J6 is set using:

```
=IF($L$1=0,H6,IF($L$1<0,1/$L$2*SIN($L$2*H6),-1/$L$2*SINH(-$L$2*H6)))
```

G) Compute the angular diameter distance, and the luminosity distance normalized by the Hubble Distance. The angular and luminosity distances are defined above on page 2. The Hubble Distance,  $D_H = c/H_0$ . Then compute the distance modulus.

**Report:** Recreate Figure 1 and the plots in Figure 4 in your report. Write a paragraph explaining the implications of the Einstein-deSitter universe. How old is this universe? How big is this universe today? Do galaxies get dimmer at all redshifts? Do galaxies appear smaller at all redshifts? Does this universe continue to expand forever or does it start to contract at some time in the future?

Debug the curvature term, check that the  $D_{pt} = D_p$  when  $(1-\Omega_0)=0$ . Check positive curvature: set  $1-\Omega_0 = -0.05$  and check that  $D_{pt} = 8281.6816$  Mpc in the first bin, where  $\log(a) = -6$ . Then set  $1-\Omega_0 = 0.05$  and check that  $D_{pt} = 8851.6134$  Mpc in the first bin, where  $\log(a) = -6$ .

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	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	Einstein-DeSitter													
2				$H_0$ (km/s/Mpc) = 70		$c$ (km/s) = 2.9979E+05			$D_H$ (Mpc) = 4.28E+03		$1-Q_0 = 0$			
3				$H_0$ (1/s) = 2.26855E-18		$c$ (Mpc/s) = 9.7156E-15			$t_H$ (sec) = 4.40811E+17		$\kappa/R_0 = 0.00E+00$			
4			$f(a) = 2/3(a^{3/2} - 1)$	emission time	time since Big Bang						$R_0 = \#DIV/0!$			
5	$\log(a)$	$a$	$H_0(t-t_0)$	$t-t_0$ (sec)	age (years)	$z(t)$	trap Area	$D_p$ (Mpc)	$D_p/D_H$	$D_{pt}$ (Mpc)	$D_{pt}/D_H$	$D_A/D_H$	$D_L/D_H$	$D_M$
6	-6	0.000001	-0.6666666666	-2.93874E+17	0.000E+00	999999	0.0992	8557.8785	1.9982	8557.8785	1.9982	1.99822E-06	1.998220.7	74.662
7	-5.99	1.023E-06	-0.6666666666	-2.93874E+17	3.273E-01	977236.221	0.1003	8557.7793	1.9982	8557.7793	1.9982	2.04474E-06	1952713.009	74.612
8	-5.98	1.047E-06	-0.6666666666	-2.93874E+17	6.660E-01	954991.586	0.1015	8557.6790	1.9982	8557.6790	1.9982	2.09235E-06	1908241.459	74.562
596	-0.1	0.7943282	-0.19470281	-8.58272E+16	6.593E+09	0.25892541	88.4074	931.5929	0.2175	931.5929	0.2175	0.172783984	0.273844161	40.346
597	-0.09	0.8128305	-0.178116978	-7.85159E+16	6.824E+09	0.23026877	89.4311	843.1854	0.1969	843.1854	0.1969	0.160029641	0.242214663	40.080
598	-0.08	0.8317638	-0.160948283	-7.09478E+16	7.064E+09	0.20226443	90.4667	753.7543	0.1760	753.7543	0.1760	0.14638856	0.211595947	39.786
599	-0.07	0.8511334	-0.143176244	-6.31137E+16	7.312E+09	0.17489755	$= (D_{600} - D_{599}) * 0.5 * (\$G\$2/\$B600 + \$G\$2/\$599)$	663.2876	0.1549	663.2876	0.1549	0.131819366	0.181961379	39.459
600	-0.06	0.8709636	-0.124779656	-5.50043E+16	7.569E+09	0.14815362	92.5739	571.7734	0.1335	571.7734	0.1335	0.11627899	0.152855563	39.086
601	-0.05	0.8912505	-0.105736572	-4.66099E+16	7.835E+09	0.12201845	93.6459	479.1994	0.1119	479.1994	0.1119	0.099722607	0.125543324	38.653
602	-0.04	0.9120106	-0.086024273	-3.79205E+16	8.111E+09	0.0964782	94.7303	385.5535	0.0900	385.5535	0.0900	0.082103567	0.098710199	38.130
603	-0.03	0.9332541	-0.065619242	-2.89257E+16	8.396E+09	0.07151931	95.8272	290.8233	0.0679	290.8233	0.0679	0.063373326	0.072762314	37.468
604	-0.02	0.9549926	-0.044497133	-1.96148E+16	8.691E+09	0.04712855	96.9368	194.9961	0.0455	194.9961	0.0455	0.043481373	0.047676377	36.550
605	-0.01	0.9772372	-0.022632747	-9.97677E+15	8.996E+09	0.02329299	98.0593	98.0593	0.0229	98.0593	0.0229	0.022375155	0.023429664	35.007
606	0	1	0	0	9.312E+09	0	99.1948	0	0.0000	0.0000	0.0000	0	0	#NUM!
607	0.01	1.023293	0.023428111	1.03274E+16	9.640E+09	-0.0227628	100.3434	99.1948	0.0232	99.1948	0.0232	0.023700967	0.022634247	34.932
608	0.02	1.0471285	0.047679537	2.10177E+16	9.978E+09	-0.0450074	101.5053	199.5381	0.0466	199.5381	0.0466	0.048786903	0.044494184	36.400
609	0.03	1.0715193	0.07278321	3.20836E+16	1.033E+10	-0.0667457	102.6807	301.0434	0.0703	301.0434	0.0703	0.075319339	0.065600402	37.243
610	0.04	1.0964782	0.098769081	4.35385E+16	1.069E+10	-0.0879892	103.8697	403.7241	0.0943	403.7241	0.0943	0.103362267	0.085972989	37.830
611	0.05	1.1220185	0.125668152	5.53959E+16	1.107E+10	-0.1087491	105.0724	507.5938	0.1185	507.5938	0.1185	0.132982234	0.105631543	38.278
612	0.06	1.1481536	0.153512514	6.767E+16	1.146E+10	-0.1290364	106.2891	612.6662	0.1431	612.6662	0.1431	0.16424844	0.124595183	38.636
613	0.07	1.1748976	0.182335387	8.03755E+16	1.186E+10	-0.148862	107.5199	718.9553	0.1679	718.9553	0.1679	0.197232836	0.142882559	38.934
614	0.08	1.2022644	0.212171159	9.35274E+16	1.228E+10	-0.1682362	108.7649	826.4752	0.1930	826.4752	0.1930	0.232010233	0.160511865	39.186
615	0.09	1.2302688	0.243055424	1.07142E+17	1.271E+10	-0.1871695	110.0243	935.2400	0.2184	935.2400	0.2184	0.268658404	0.177500848	39.405
616	0.1	1.2589254	0.27502503	1.21234E+17	1.315E+10	-0.2056718	111.2983	1045.2644	0.2441	1045.2644	0.2441	0.307258202	0.193866819	39.596
617	0.11	1.2907696	0.300110116	1.35929E+17	1.362E+10	-0.2253376	112.5815	1152.6733	0.2691	1152.6733	0.2691	0.347036327	0.219926222	39.762

Figure 3: Derived observable quantities in the Einstein-DeSitter universe.

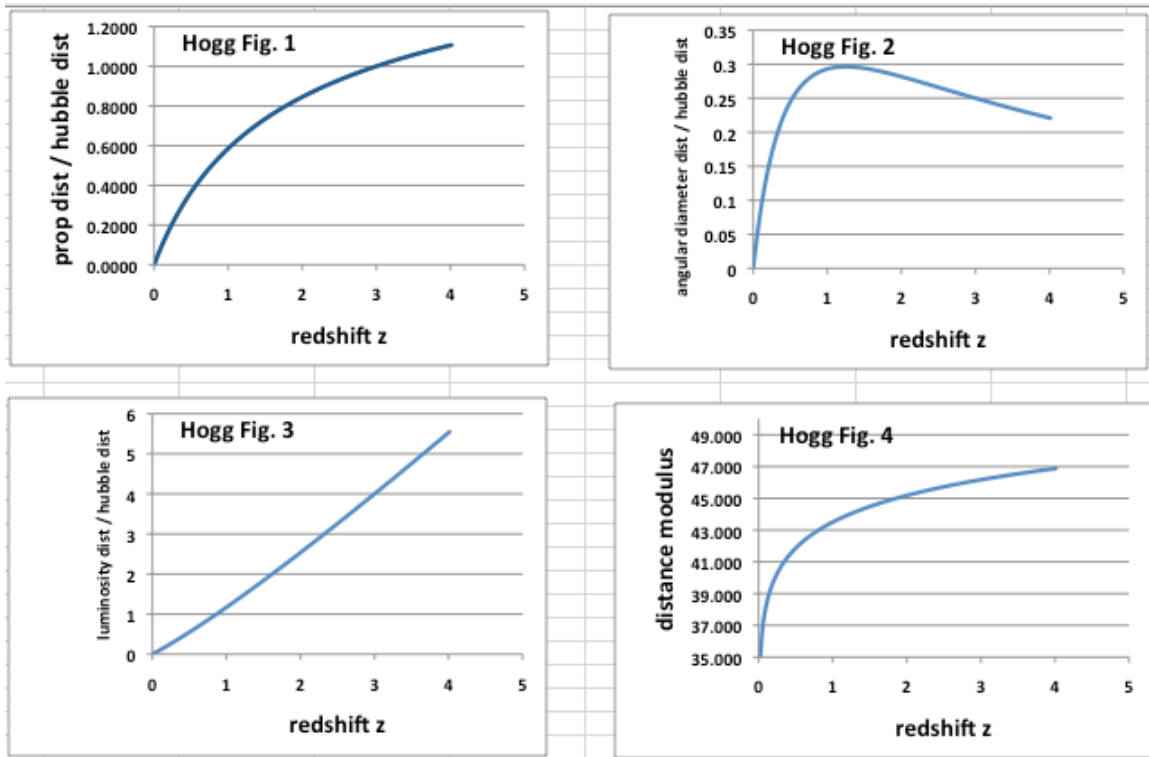


Figure 4: Plots of measurable parameters. These are recreations of Figures in reference [2] by David Hogg.

## Part 2 – General Solution to the Friedman Equation

Barbara Ryden derives the Friedman Equation, the fluid equation, and the equation of state in chapter 4. Together, they are solved in chapters 5 & 6. We will concentrate here on the general solution: (Ryden Eqn. 6.8),

$$\int_1^a \frac{da'}{\sqrt{\frac{\Omega_{r,0}}{a'^2} + \frac{\Omega_{m,0}}{a'} + \Omega_{\Lambda,0}a'^2 + (1 - \Omega_0)}} = H_0 \int_{t_0}^t dt' \quad 9$$

where the history of the universe is embodied in the time,  $t$ , and the expansion scale factor,  $a$  that is governed by the  $\Omega$  parameters measured at their current epoch. This equation includes the radiation, matter, and dark energy density as well as the resulting curvature term  $(1 - \Omega_0)$ . The right-hand side is easily integrated giving the familiar  $H_0 \int_{t_0}^t dt' = H_0(t_e - t_0)$ . The left-hand side is more complicated and has no nice analytical solution. Furthermore, it can't be inverted into the form  $a(t) = f(t)$  as before. Be sure that you understand the derivation of Eqn. 9 and what it means. Once you trust the physics behind the equation, you can use it to compute interesting facts about the universe.

**General Instructions:** Start with the same  $\log(a)$  array as you did previously. Compute  $a$ , and  $z$  as before. Next numerically integrate the left-hand-side of Eqn. 6

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and set it equal to  $H_0(t-t_0)$ . Just like before, you'll have to be careful with the integration limits. Set  $a=1$  when  $t=t_0$  and then integrate backward to  $t$  in the past and forward to  $t$  in the future. Check that you get the results shown in the spreadsheet in column F of Figure 6 for the Ryden's Benchmark cosmology. Recreate Figure 7 below.

**Excel Instructions:** Open a new sheet (tab at the bottom) and copy  $\log(a)$  into it from the previous sheet. Then compute  $a$  and  $z$  from  $\log(a)$ . Setup the parameters needed for the benchmark cosmology. Now we need to compute the integral. The integrand is:

$$f(a') = \frac{1}{\sqrt{\Omega_{r,0} a'^2 + \frac{\Omega_{m,0}}{a'} + \Omega_{\Lambda,0} a'^2 + (1 - \Omega_0)}} \quad 10$$

and we compute the area under this function as we did before. Compute Column E. Finally, again being careful with the integration limits, we add the areas into the past and again into the future to find  $H_0(t-t_0)$ . Set  $H_0(t-t_0)=0$  at  $t_e=t_0$  then add the area of the trapezoids up and down in the table from this point.

	A	B	C	D	E	F
1	General Solution to Friedman Equation				Parameters	
2					Omega_M	0.3
3					Omega_r	8.40E-05
4					Omega_Lambda	0.7
5					Omega_tot	1
6					Ho	70
7						
8						
9						
10	log(a)	a	z	f(a)	trap. area	Ho(t-to)
11	-6	0.000001	999999	1.09E-04	2.57E-12	-9.64E-01
12	-5.99	1.0233E-06	977236.221	1.11E-04	2.69E-12	-9.64E-01
13	-5.98	1.0471E-06	954991.586	1.14E-04	2.81E-12	-9.64E-01
14	-5.97	1.0715E-06	933253.301	1.17E-04	2.95E-12	-9.64E-01
15	-5.96	1.0965E-06	912009.839	1.19E-04	3.09E-12	-9.64E-01
16	-5.95	1.122E-06	891249.938	1.22E-04	3.23E-12	-9.64E-01
601	-0.1	0.79432823	0.25892541	1.10E+00	2.04E-02	-2.17E-01
602	-0.09	0.81283052	0.23026877	1.10E+00	2.07E-02	-1.96E-01
603	-0.08	0.83176377	0.20226443	1.09E+00	2.10E-02	-1.76E-01
604	-0.07	0.85113804	0.17489755	1.08E+00	=(B605-B6)4)*0.5*(D605+D604)	
605	-0.06	0.87096359	0.14815362	1.07E+00	2.16E-02	-1.34E-01
606	-0.05	0.89125094	0.12201845	1.06E+00	2.19E-02	-1.12E-01
607	-0.04	0.91201084	0.0964782	1.05E+00	2.21E-02	-9.01E-02
608	-0.03	0.9332543	0.07151931	1.04E+00	2.24E-02	-6.80E-02
609	-0.02	0.95499259	0.04712855	1.02E+00	2.27E-02	-4.56E-02
610	-0.01	0.97723722	0.02329299	1.01E+00	2.29E-02	-2.29E-02
611	0	1	0	1.00E+00	2.31E-02	0.00E+00
612	0.01	1.02329299	-0.0227628	9.87E-01	2.34E-02	2.31E-02
613	0.02	1.04712855	-0.0450074	9.74E-01	2.36E-02	4.65E-02
614	0.03	1.07151931	-0.0667457	9.61E-01	2.38E-02	7.01E-02
615	0.04	1.0964782	-0.0879892	9.47E-01	2.40E-02	9.39E-02
616	0.05	1.12201845	-0.1087491	9.33E-01	2.42E-02	1.18E-01
617	0.06	1.14815362	-0.1290364	9.19E-01	2.44E-02	1.42E-01
618	0.07	1.17489755	-0.148862	9.05E-01	2.46E-02	1.67E-01
619	0.08	1.20226443	-0.1682362	8.90E-01	2.47E-02	1.91E-01
620	0.09	1.23026877	-0.1871695	8.76E-01	2.49E-02	2.16E-01
621	0.1	1.25892541	-0.2056718	8.61E-01	2.50E-02	2.41E-01

Figure 5: Spreadsheet showing the general solution to the Friedman Equation for the Benchmark Model.



## Expansion History of the Universe

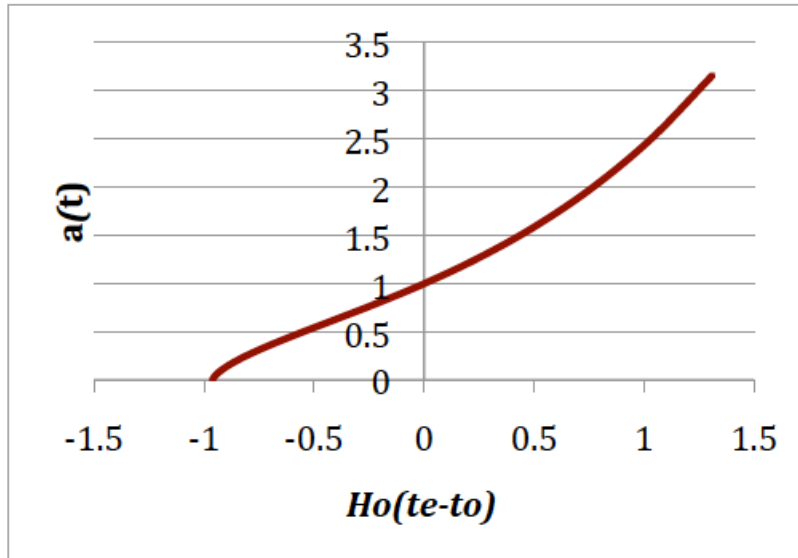


Figure 6: Scale factor as a function of time for the Benchmark Model.

**Report:** Include your recreations of Figure 1 and Figure 6 in your report. How do they differ from each other? What is the significance of the differences?

### Part 3—Comparing the Cosmologies

Create another “Arbitrary” spreadsheet by copying the Einstein-deSitter spreadsheet into a new spreadsheet and relabeling the  $H_0(t-t_0)$  column as shown. Plots don’t copy correctly so don’t bring them along. The numbers in the columns can be copied without changing anything! Now let’s hook the new spreadsheet up to the General Solution. Set the  $H_0(t-t_0)$  column equal to the general solution from your previous spreadsheet as shown below. Set  $H_0$  to the value in the other spreadsheet as well. Set  $1-\Omega_0$  from the other spreadsheet too. DON’T CHANGE ANY EQUATIONS! THEY ALREADY WORK! In this part, we’re just hooking up the general solution to the computation of the observables.

	A	B	C	D	E	F
1	Arbitrary		Ho (km/s/Mpc) = 70			c (k
2						c (Mpc
3						
4			f(a) = arbitrary	emission time	time since Big Bang	
5	log(a)	a	Ho(t-to)	t-to (sec)	age (years)	
6	-6	0.000001	-9.64E-01	-4.26773E+17	0.000E+00	
7	-5.99	1.0233E-06	-9.64E-01	-4.26773E+17	3.604E-02	977
8	-5.98	1.0471E-06	-9.64E-01	-4.26773E+17	7.378E-02	954
9	-5.97	1.0715E-06	-9.64E-01	-4.26773E+17	1.133E-01	9332
10	-5.96	1.0965E-06	='General solution to Friedman'!F15			9120
11	-5.95	1.122E-06	-9.64E-01	-4.26773E+17	1.980E-01	8912
12	-5.94	1.1482E-06	-9.64E-01	-4.26773E+17	2.434E-01	87

Figure 7: Computation for an arbitrary cosmology. Except for Column C and cell E1, the rest of the spreadsheet should be unchanged from before.

**Report:** The following can be done without modifying the columns of your spreadsheet. From here on out you should just be changing the parameters in the spreadsheet with the General Solution.

- A. Verify that you compute the same age of the universe that Ryden computes in Table 6.2 for the Benchmark cosmology. Then recreate the solid curve in the top plot of Ryden Fig. 6.6. We're going to explore several different cosmologies.
- B. Make a table with the following columns: name of cosmology,  $H_0$ ,  $\Omega_{r,0}$ ,  $\Omega_{m,0}$ ,  $\Omega_{\Lambda,0}$ ,  $\Omega_0$ ,  $\kappa$ ,  $R_0$ , age of the universe, distance to a galaxy at the edge of the visible universe, and eventual fate (expand forever or collapse?). Fill it in for the Benchmark cosmology.
- C. Repeat the process to make the other curves in Ryden Fig. 6.6. Fill in the table for the  $\Lambda$ -Only and Matter-Only universes. The Matter-Only universe is the Einstein-deSitter universe. Does everything match your original spreadsheet for the Matter-only universe?
- D. Another interesting universe is the Low-Density, Matter-Only universe. With so much empty space in the universe, lets investigate a Matter-Only universe with  $\Omega_o = \Omega_{m,o} = 0.05$ . What is the age of this universe and the distance to a galaxy at the edge? Why is this universe philosophically attractive? In the next few weeks we will find out that this intellectually appealing universe is not the one that we actually live in. We are going to look at observations and see that they differ from this universe. How does the proper distance differ from those in Ryden's Figure 6.6?
- E. Where were you in 1998? Until 1998 it was believed that  $\Omega_{\Lambda,0}=0$ , and astrophysicists focused their research on measuring  $\Omega_0$  to determine the curvature of the universe. The scale factor  $a(t)$  was so poorly known that the uncertainty in the age of the universe was about 50%, i.e. 12.6 billion years. Today, the age of the universe is known to be  $13.75 \pm 0.11$  billion years. This precision is better than 1% and represents a giant leap in knowledge about the origin of the universe. In just the past 12 years, the  $\Omega$  parameters have been measured with extreme precision. The latest parameters can be found in Table 8 on page 39 of the WMAP paper published in January of 2010 [3]. Use the parameters in the WMAP+BAO+ $H_0$  column to compute the age of the universe. WMAP did not measure  $\Omega_{r,0}$  which was measured by the earlier COBE satellite mission. Feel free to use the value  $\Omega_{r,0} = 8.40E-5$  for this parameter. Add the WMAP cosmology to your table. How well did you do? Does your value agree with the age in the paper?
- F. Ryden's Benchmark model serves the important purpose of computing the expansion history pretty well in light of the rapid changes in the field. It is impossible to produce new editions of the book every time a paper is published. With this code, you can always use the most current values as they become available. Please comment on the difference in age and size between the Benchmark and WMAP cosmologies. Can you expect the computations in the textbook to be valid to a few percent?
- G. Tell me about the WMAP universe. How old is it? How big is the visible universe today? Does this universe continue to expand forever or does it start to contract

## Expansion History of the Universe

at some time in the future? Do galaxies get dimmer at all redshifts? Do galaxies appear smaller at all redshifts?

Please include the tables and requested plots in your report. Write explanations as needed to answer the various questions.

### References:

- B. Ryden, *Introduction to Cosmology*, Addison Wesley, (2004).
- D.W. Hogg, *Distance measures in cosmology*, (2000), <http://arxiv.org/abs/astro-ph/9905116>.
- WMAP collaboration, *Seven-Year Wilkinson Microwave Anisotropy Probe Observations: Sky Maps, Systematic Errors, and Basic Results*, Jarosik, et.al., (2011) *ApJS*, **192**, 14; <http://arxiv.org/abs/1001.4744>.