# Initial Tests for an Analog Continuum Correlator for CMB Interferometry Huan T. Tran

advisor: Lyman Page

#### Abstract

We are proposing to build a high frequency interferometer to study the CMB anisotropy. In support of our proposal, we have tested a high bandwidth (5GHz) multiplier for use in an analog complex correlator. A lag autocorrelator spectrometer was built and tested up to 2.6 GHz.

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### 1. Introduction

The last half decade has seen the beginning of many interferometry programs to study the CMB. In the continuing effort to probe smaller angular scales, interferometers are becoming the natural choice for ground based observations. Interferometry has already proven it's usefulness in longer wavelength radio astronomy, and the step to microwaves is in many ways just a reapplication of many proven but updated technologies and ideas. The main technological advance that has allowed microwave interferometry is high sensitivity, high frequency coherent detectors. There are two major technologies involved: HEMT amplifiers and SIS junction mixers. HEMT (High Electron Mobility Transistor) Amplifiers have been used with much success at lower frequencies and in recent years have been pushed to higher ~90GHz frequencies by the use of exotic ultrafast semiconductors. SIS (Superconductor Insulator Superconductor) junction mixers have made interferometry possible at wavelengths into the THz region. We have used both technologies as square law detectors to probe CMB anisotropy at <sup>1</sup>/<sub>4</sub> degree scales. With interferometry, we hope to push the resolution to smaller scales by an order of magnitude.

Most CMB anisotropy studies are concerned with the angular power spectrum, and interferometers can measure this directly by outputting the spatial Fourier transform of the sky. Interferometers also only correlate signals that are common to pairs of receivers. Hence interferometers are insensitive to correlated receiver noise. Also, since the beams of adjacent dishes do not intersect for a  $\sim 100$  meters away from the telescope, they are also insensitive to ground clutter. The atmosphere contributes a secant gradient from the zenith to the ground. Interferometers are only sensitive to a chosen range of spatial frequencies, and these are usually chosen to be very high, which renders them insensitive to slowly varying sky gradients.

Although it is not the primary goal of most experiments, a map of the anisotropy can be produced by inverting the output. As a proof of concept, the CAT telescope (Jones 1996) has already produced a map at 13-17 Ghz and ½ degree resolution. Three more CMB

interferometers are planned to begin operation in the next 2 years: VSA, DASI, and CBI<sup>1</sup>. All have many elements (13-15) and work below 35 GHz and hence rely heavily on HEMT technology. We are in the initial planning stages to build a much higher frequency (150 GHz) interferometer, which will employ SIS mixers. A variety of frequencies is essential in studying the CMB to eliminate galactic foreground contamination.

An outline of this paper is as follows. We will begin with the analysis of a simple two element interferometer and show that the output is the Fourier transform of the intensity on the sky. We will follow with a discussion of the design of a complex correlator and end with a description and results of a test of a high bandwidth analog multiplier. 2.Analysis of a Simple Interferometer



output

Figure 1: A simplified diagram of a 2 element 1 dimensional interferometer including geometrical delay and correlator

Figure 1 shows a schematic of one baseline on a simplified interferometer. Two antennas are separated by a distance D. Cosmic signal from  $d\theta$  and direction  $\theta$  enters each antenna as shown. We are making the approximation that the sources are at infinite distance

<sup>&</sup>lt;sup>1</sup> VSA: Very Small Array see Jones 1996 or

http://www.mrao.cam.ac.uk/telescopes/cat/vsa.html

DASI: Degree Angular Scale Interferometer see http://astro.uchicago.edu/dasi/

CBI: Cosmic Background Imager see http://astro.caltech.edu/~tjp/CBI/index.html

allowing us to assume that the wavefront is essentially plane and the two rays drawn in figure 1 can be considered parallel. We also assume that the sizes of the dishes are small relative to the separation D and the response of the two antennas is omnidirectional and proportional to the electric field strength  $E(\theta,t)$ . After coupling to the antennas and waveguide, the signals travel through symmetric paths to the correlator, where they are multiplied and integrated. For suitably narrow bandpass, the signals can be considered sinusoidal with frequency v and the output of the multiplier can be written as:

$$F(\theta, t) = E^{2}(\theta)\sin(2\pi\nu t + \phi(\theta))\sin(2\pi\nu(t - \tau) + \phi(\theta))$$
(1)

where  $\tau = \frac{D}{c} \sin \theta$ , the geometric delay and  $\phi(\theta)$  is a random function. The randomness of  $\phi(\theta)$  implies that the sky is spatially incoherent. For simplicity we will take  $\phi(\theta)$  for this direction to be zero. The above equation can be re-written as:

$$F(\theta, t) = \frac{E^2(\theta)}{2} \left( \cos(2\pi\nu\tau) - \cos(4\pi\nu t)\cos(2\pi\nu\tau) - \sin(4\pi\nu t)\sin(2\pi\nu\tau) \right)$$
(2)

For a single d $\theta$  on the sky,  $\tau = \frac{D}{c} \sin \theta$  varies slowly as the sky rotates. The last two terms vary much faster than the first, with on the order of 150 GHz or, if the signals are mixed down (explained below), v is on the order of 3 GHz. We can either integrate or filter out the last two terms and ignore factors of 2 which leaves us with the response of the interferometer:

$$F(\theta) = E^{2}(\theta) \cos\left(\frac{2\pi D}{\lambda}\sin\theta\right); \quad \lambda = \frac{c}{\nu}$$
(3)

Real antennas have a finite response to off axis radiation. The maximum angle is in practice of order 1 degree. We can therefore make the small angle approximation. In addition, since the we assumed that the sky was spatially incoherent, we can integrate independently the response from different points on the sky to give the full sky response.

$$\int F(\theta) d\theta = V(u) = \int E^{2}(\theta) \cos(2\pi u\theta) d\theta$$
(4)
Where we define  $u = \frac{D}{\lambda}$ .

 $V(\theta)$  is called the Visibility function which is here identified as the real Fourier transform of  $E^2(\theta)$  with conjugate variables u and  $\theta$ . As we can see, the simple interferometer with one baseline will output the value of the real transform at  $u = D/\lambda$ . This of course, is not the entire picture. To reconstruct a 2- dimensional  $E^2(\theta)$  we need many baselines of varying u and in the transverse orientation, v. The cosine transform only encodes information about the symmetric components of  $E^2(\theta)$ . Since  $E^2(\theta)$  is not in general symmetric about  $\theta = 0$ , information is also encoded in the sine transform. In equivalent language, we need both amplitude and phase information, which requires both the real and the complex part of the visibility.

Real-world interferometers have a finite sized dish, sometimes comparable to D the baseline. The effect of this is to smear out the values of u, allowing us to only measure an average value from some range of u's. In studying CMB anisotropy, we are only interested in the power spectrum at a limited range of spatial frequencies, so a single length baseline is adequate. A better way to spend extra baselines is by arranging them so as to maximize 2d coverage. The complex part of the visibility can be measured by a complex correlator, which will be explained in the next section.

#### 3. Continuum Complex Correlator

Continuum Complex correlators are used to measure the complex visibility instantaneously over a range of frequencies. The design has been around for some time, and these types of correlators have been used at many observatories, including Owens Valley (Padin 1994) and will be used on all of the planned CMB interferometers.



Fig 2: An outline of a complex correlator. The main difference is an addition of a quadrature network or Hilbert transformer which allows the recovery of the imaginary visibility

Figure 2 is an outline of a complex correlator. The bottom branch of the correlator is equivalent to the simple correlator discussed in the previous section. The addition of the quadrature network in the top branch makes this a complex correlator. The Quadrature network delays the signal in one of the arms by 90 degrees. This is often known as a Hilbert transform and is not equivalent to a time lag for finite bandpass. The effect is to add a phase shift of  $\pi/2$  in one of the arguments in equation 1:

$$F(\theta, t) = E^{2}(\theta) \sin\left(2\pi v t + \frac{\pi}{2}\right) \sin\left(2\pi v (t - \tau)\right)$$
(5)

The result is that the cosine in equation 4 is replaced by a sine and we have the complex transform:

$$\operatorname{Im} V(u) = \int E^{2}(\theta) \sin(2\pi u \theta) d\theta$$
(6)

A complex correlator allows us to measure both the real and imaginary components of the complex Visibility V(u) simultaneously and the output of the interferometer can be written as:

$$\mathbf{V}(\mathbf{u}) = \int \mathbf{E}^2(\mathbf{\theta}) \mathbf{e}^{\mathbf{i}2\pi\mathbf{u}\mathbf{\theta}} \mathbf{d\mathbf{\theta}}$$
(7)

#### 4. Some Practical considerations

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A few more problematic assumptions made above need to be addressed. Firstly, the frequencies that we are interested in observing are very high,  $\sim 150$  GHz. It is extremely difficult to find general equipment that functions at these frequencies. Radio astronomers are accustomed to mixing down the signals to more manageable frequencies. At 150 GHz, the current state of the art is SIS junction mixers.



Figure 3: An outline of a simple interferometer with mixers, amplifiers, phase locking and phase switching. The matching delay is used to match the path lengths from the correlator to the antennas.

Mixers require a reference LO (local oscillator) signal. The incoming RF signal is mixed with the LO signal and is downconverted to the IF (intermediate frequency). Waveguide used at 150 GHz is lossy. To keep power requirements low, a separate LO is placed in

close proximity to each mixer. To maintain coherence, the LO's must be phase locked together. The subject of phase locking will be discussed by Dorwart (1998). An added advantage of using separate LO's is that we can deal with internal offsets by implementing a phase switch in one of the LO's. This would periodically switch one of the inputs to the correlator by 180 degrees which switches the output of the multiplier by - 1. If the phase switch signal is then demultiplexed by the correlator before integration, the internal offset can be removed.

A second assumption has to do with the finite bandpass. To achieve higher sensitivities we would like to allow a broader range of frequencies though the system. Typically, the bandwidth,  $\Delta v$ , of SIS mixers are ~ 2 GHz. Compared to 150Ghz, this still leaves  $\Delta v/v$  small and most of our assumptions are still valid. The effect of a finite bandwidth is to wash out the visibility at large  $\theta$ . Since we are limiting ourselves to small  $\theta$  already, this is not an issue. When the signal is mixed down, however,  $\Delta v/v$  becomes much larger because the bandwidth is preserved but the center frequency is much lower. The upshot is that the correlator must be able to handle a large bandwidth, at least as large as the SIS, ~ 2GHz. Correlators that correlate over a finite bandwidth are known as continuum correlators. The main technological problem areas are the quadrature networks and multipliers. The former is sometimes called a 90 degree phase hybrid and is commercially available in our frequencies of interest as a separate component which may be plugged into our system. The alternative is to design a quadrature network into the microstrip circuitry of the correlator itself. This possibility, however, requires more testing and sophisticated design. The multiplier will be the focus of the rest of this paper.

## 5. Multiplier Tests and Spectrometer

At the heart of a correlator is the analog multiplier We have decided to use the commercial HP-IAM81008 MMIC (Monolithic Microwave Integrated Circuit) active mixers as the multiplier. New manufacturing processes have led to these inexpensive, high sensitivity broadband multipliers. The chips use a standard Gilbert cell multiplier (Gilbert 1968) and have been well tested in correlators (Harris et al 1998).

As an exercise, we have used the MMIC chip to make a spectrometer. The output of the spectrometer is the full autocorrelation function:

$$R(\tau) = \left\langle E(t)E(t-\tau) \right\rangle \tag{8}$$

Where E(t) could be the output from a single receiver split into two arms and  $\tau$  is a time delay inserted into one of the arms. The brackets denote averaging over time t.

By the Weiner-Khinchin theorem we know that the Autocorrelation and the Power Spectral Density form a Fourier transform pair:

 $R(\tau) \underset{F}{\longleftrightarrow} PSD(\nu) \tag{9}$ 

Obtaining the PSD from the autocorrelation function is just a matter of taking the Fourier transform. A schematic of our spectrometer, commonly known as a lag spectrometer, appears below.



Figure 4: An outline of a lag spectrometer, the device used to test the multiplier chip. The transmission lines were mostly 50 ohm coax with a transition to microstrip near the multiplier. The power divider was a standard Wilkinson junction and the modulator had a nominal 1-2 Ghz operating range full vector capabilities. DC blocking capacitors were used to isolate the cell form the sweeper and signal analyzer.

The input signal was provided by a 0- 2.6 GHz sweeper. The variable time lag was introduced by 5 line stretchers in series. From equation 3 we can see that the output of the multiplier is essentially at DC. The HP chip, however, is not stable when operating near DC. The solution is to modulate one of the inputs. This is equivalent to the phase switching technique described above, except the modulation is mixed in directly instead of through a local oscillator. 20 kHz was chosen as a relatively clean area of Jadwin Hall. instead of demodulating the signal after the multiplier, the signal is merely fed into a signal analyzer and the height of the peak at 20 kHz is taken as the output of the spectrometer. The results of a few different input frequencies are shown below.



#### correlation of sweeper output of freq=1 GHz



#### correlation of sweeper output of freq=2.6 GHz



Figure 5: Output from the lag spectrometer. Each point represents a different setting of the line stretchers, or a different relative lag between arms in the spectrometer. A Fourier transform of this output gives the PSD of the sweeper. The relative frequencies as fit from the above plots are not in the correct ratio given the sweeper setting because there is considerable harmonic leaking.

Even though the line stretchers could only cover about  $\lambda/4$  of the correlation function, we can already see that the autocorrelation varies faster for higher frequencies and that the function is has the correct waveform from the sinusoidal fits. The fitted frequencies are not in the correct ratio. This is because the sweeper has many higher order harmonics that push the center frequency higher. To recover the entire spectrum with the harmonics resolved, many more line stretchers are required.

### 5. Conclusion and future

We have demonstrated that the HP-IAM81002 can produce correlation at 2.6 GHz. The reason it was not tested higher was because the modulator was only rated up to 2 Ghz. Harris et al 1998 have shown that the chip will operate up to 5GHz. We plan on pushing our bandwidth higher with better microstrip circuitry and higher frequency sweepers and modulators. The current plan is to design and build an entire complex correlator using these chips and implementing as many components as possible in microstrip.

### 6. References

Dorwart. W.B. 1998, Generals project May, Princeton University Gilbert , B., 1968, IEEE J. Solid-State Circuits **3**, 365-373 Harris, A. I., Isaak, K.G., Zmuidzinas, J. 1998 Proc. Of SPIE, in press Jones, M. E. 1996 astro-ph / 9611212 Padin S., 1994, IEEE Trans. On Instrumentation and Measurement **43** 782-785