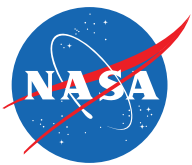


White Noise study for the CMB Probe, TDM option

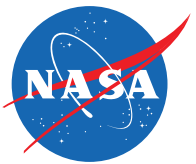
Roger O'Brient

Jet Propulsion Laboratory,
California Institute of Technology

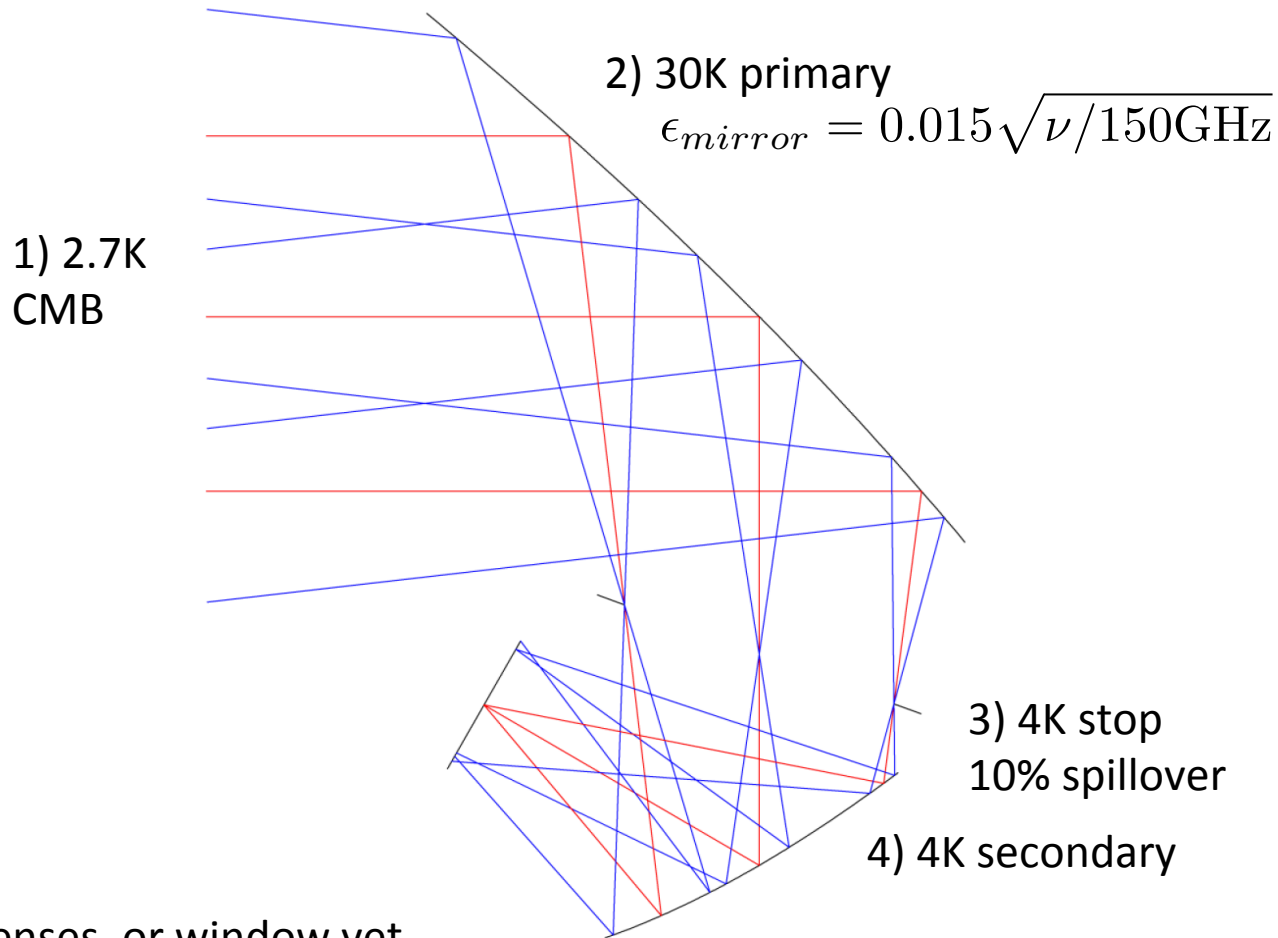


Outline

1. Optical Loading
2. Bolometer Gs
3. NEP & NEI vs audio frequency
4. Aliasing
5. Science channel white noise NEP totals
6. Vetting of model against BICEP2/Keck
7. Cost of Aliasing

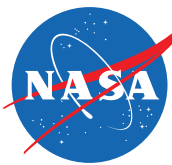


Assumptions for Optical Loading¹



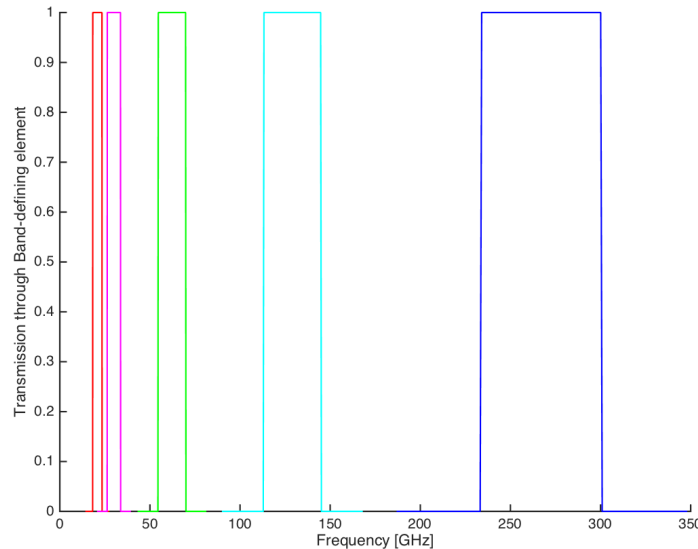
- No filters, lenses, or window yet

$$P_i = \int dA d\Omega d\nu \frac{h\nu^3/c^2}{e^{h\nu/k_B T_i} - 1} [\eta_{det} B(\nu)] \epsilon_i(\nu) \prod_{j>i} (1 - \epsilon_j(\nu))$$



Focal Plane features for Loading Calc¹

- 25% bandwidth, top-hat profiles
- Five illustrative bands (21 total):



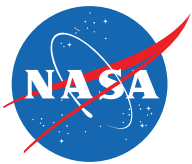
- Antenna-coupled: single moded $A\Omega = \lambda^2$
- Detector efficiency $\eta_{\text{det}} = 0.7$
- Fixed beam-width wrt frequency: fixed 10% spillover



Optical Loading

Channel	Cent_Freq [GHz]	P_cmb	P_mirror1	P_stop	P_mirror2	P_total	T_rj [K]
1	20.8	0.1000	0.0073	0.0174	0.0010	0.1257	2.50
2	25.0	0.1152	0.0096	0.0203	0.0013	0.1464	2.42
3	30.0	0.1317	0.0126	0.0236	0.0016	0.1695	2.34
4	36.0	0.1510	0.0166	0.0276	0.0020	0.1972	2.27
5	43.2	0.1664	0.0214	0.0312	0.0025	0.2215	2.12
6	51.8	0.1875	0.0287	0.0362	0.0032	0.2556	2.04
7	62.2	0.1971	0.0364	0.0395	0.0039	0.2769	1.84
8	74.6	0.2072	0.0473	0.0436	0.0047	0.3027	1.68
9	89.6	0.2112	0.0614	0.0471	0.0055	0.3252	1.50
10	107.5	0.2093	0.0805	0.0502	0.0064	0.3465	1.33
11	129.0	0.1934	0.1025	0.0508	0.0071	0.3538	1.14
12	154.8	0.1698	0.1317	0.0501	0.0077	0.3593	0.96
13	185.8	0.1381	0.1684	0.0471	0.0080	0.3615	0.81
14	222.9	0.1020	0.2142	0.0416	0.0077	0.3655	0.68
15	267.5	0.0682	0.2739	0.0347	0.0070	0.3839	0.59
16	321.0	0.0380	0.3389	0.0255	0.0057	0.4081	0.53
17	385.2	0.0186	0.4309	0.0174	0.0042	0.4712	0.51
18	462.2	0.0072	0.5210	0.0100	0.0027	0.5408	0.48
19	554.7	0.0021	0.6204	0.0048	0.0014	0.6286	0.47
20	665.6	0.0005	0.7336	0.0019	0.0006	0.7365	0.46
21	798.7	0.0001	0.8468	0.0006	0.0002	0.8477	0.44

- All power units are pW
- P_total sums contributions from 4 sources
- $T_{RJ} = P_{tot} / (k_B \eta \Delta \nu)$



Bolometer Properties: G & τ

- Bolometer saturates at a safety factor of SF=2.0 above total loading² $P_{sat} = P_{tot} + P_{Joule} = SF * P_{tot}$

- Thermal Conductivity at Transition¹:

$$P_{sat} = G_c T_c \frac{1 - (T_o/T_c)^{\beta+1}}{\beta + 1}$$

- Can rescale to other temps (e.g. Bath T_o , 450mK)

$$G_* = G_c (T_*/T_c)^\beta$$

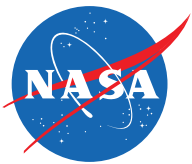
- $\beta \sim 2$ because leg thickness is less than a phonon wavelength^{2,3}

- Thermal Time-constant, sans feedback⁴: $\tau_o = C/G$

- Loop gain from electrothermal feedback^{3,4} $\mathcal{L} = \frac{\alpha P_{Joule}}{G_c T_c} \sim 17$

- α describes changes in TES R vs T - typically ~ 100

- Thermal Time-constant, with loop gain⁴: $\tau = \frac{\tau_o}{\mathcal{L} - 1}$



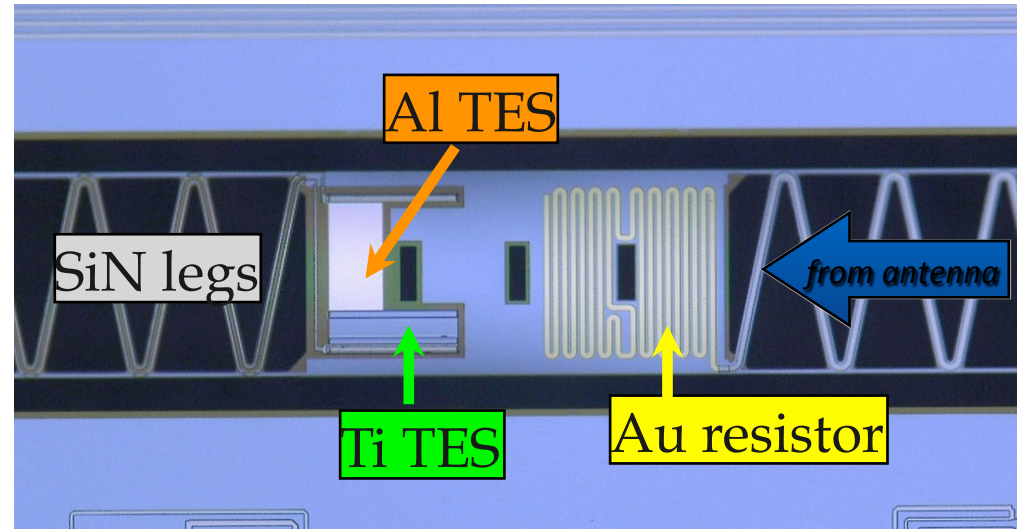
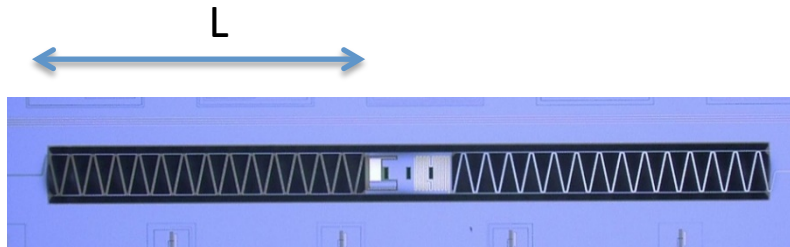
Bolometer Properties for $T_o=250\text{mK}$

Freq [GHz]	P_sat [pW]	Go [pW/K]	Gc [pW/K]	G450 [pW/K]	tau [ms]
20.8	0.30	0.54	2.10	1.73	59.21
25.0	0.35	0.62	2.44	2.02	50.82
30.0	0.41	0.72	2.83	2.34	43.90
36.0	0.47	0.84	3.29	2.72	37.72
43.2	0.53	0.94	3.70	3.06	33.58
51.8	0.61	1.09	4.27	3.53	29.10
62.2	0.66	1.18	4.62	3.82	26.87
74.6	0.73	1.29	5.05	4.18	24.57
89.6	0.78	1.38	5.43	4.49	22.88
107.5	0.83	1.48	5.79	4.78	21.47
129.0	0.85	1.51	5.91	4.88	21.03
154.8	0.86	1.53	6.00	4.96	20.71
185.8	0.87	1.54	6.04	4.99	20.58
222.9	0.88	1.56	6.10	5.04	20.35
267.5	0.92	1.63	6.41	5.30	19.38
321.0	0.98	1.74	6.81	5.63	18.23
385.2	1.13	2.01	7.87	6.50	15.79
462.2	1.30	2.30	9.03	7.46	13.76
554.7	1.51	2.68	10.50	8.67	11.84
665.6	1.77	3.14	12.30	10.16	10.10
798.7	2.03	3.61	14.15	11.70	8.78

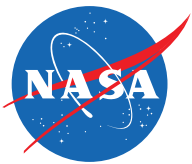
- $T_o=250\text{mK}$, $T_c=500\text{mK}$ (Ti)
- These Gs will be easier to achieve with low $T_o \sim 100\text{mK}$ than 250mK (see next slide)
- I've presumed $C \sim 1\text{pJ/K}$, achievable with few μm thick Gold at $T_c \sim 500\text{mK}$. By thinning/omitting the gold, we can speed some of these up. (Or drop the T_c)



Bolometer Design Considerations



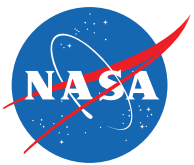
- $G_c \sim 2 \text{pW/K}$ ($T_c \sim 500 \text{mK}$) is **challenging** for existing JPL recipes.
- Scaling from lowest JPL achieved $\sim 15 \text{pW/K}$ (SPIDER 95GHz⁵) detector, a 20GHz CMB-probe would use $L \sim 10 \text{mm}$ long leg, zig-zagged to be $\sim 40 \text{mm}$ long.
- Largest device we have made has $L \sim 1 \text{mm}$ long legs
- If we change bath temp from 250mK \rightarrow 100mK, then G drops by $(100/250)^2$ for a given geometry.
- So we would need $L \sim 1.5 \text{mm}$ to achieve $G_c \sim 2 \text{pW/K}$ or $G_o \sim 0.54 \text{pW/K}$.



Bolometer Properties for $T_o=100\text{mK}$

Freq [GHz]	P_sat [pW]	G0 [pW/K]	Gc [pW/K]	tau [ms]
20.8	0.30	1.29	5.17	24.16
25.0	0.35	1.51	6.02	20.74
30.0	0.41	1.74	6.97	17.91
36.0	0.47	2.03	8.11	15.39
43.2	0.53	2.28	9.11	13.70
51.8	0.61	2.63	10.52	11.87
62.2	0.66	2.85	11.39	10.96
74.6	0.73	3.11	12.46	10.03
89.6	0.78	3.34	13.38	9.33
107.5	0.83	3.56	14.26	8.76
129.0	0.85	3.64	14.56	8.58
154.8	0.86	3.70	14.78	8.45
185.8	0.87	3.72	14.87	8.40
222.9	0.88	3.76	15.04	8.30
267.5	0.92	3.95	15.79	7.91
321.0	0.98	4.20	16.79	7.44
385.2	1.13	4.85	19.38	6.44
462.2	1.30	5.56	22.25	5.61
554.7	1.51	6.47	25.86	4.83
665.6	1.77	7.58	30.30	4.12
798.7	2.03	8.72	34.88	3.58

- $T_o=100\text{mK}$, $T_c=200\text{mK}$ (AlMn, Hf)
- Strong driver for lower temperature



Detector NEP, no aliasing yet^{1,6}

$$NEP_{\gamma,shot}^2 = 2h\nu P_{opt}$$

$$NEP_{\gamma,bose}^2 = 2P_{opt}^2 / \sqrt{\Delta\nu}$$

Photon
Noise

• ν refers to optical frequencies: 10GHz-1THz

$$NEP_{TES}^2 = 4K_B T_c R_o I_o^2 \frac{1}{\mathcal{L}} (1 + (2\pi f \tau_o)^2)$$

$$NEP_{shunt}^2 = 4k_B T_{sh} R_{sh} I_o^2 \frac{(\mathcal{L} - 1)^2}{\mathcal{L}^2} (1 + (2\pi f \tau)^2)$$

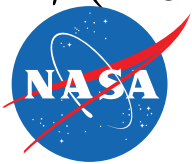
Johnson
Noise

• f refers to audio time-stream frequencies: 10mHz-10kHz

$$NEP_G^2 = 4\gamma(T_o, T_c) k_B G_c T_c^2$$

$$\gamma(T_o, T_c) \equiv \frac{1 + \beta}{2\beta + 3} \frac{(T_o/T_c)^{3+2\beta} - 1}{(T_o/T_c)^{1+\beta} - 1}$$

Phonon
(Leg)
Noise



Time Constants of Modes⁶

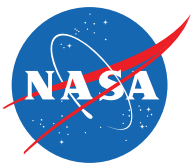
- Detector performance governed by linearized 1st-order differential equations for $(\delta I, \delta T)$
- Modes have eigenvalues of:

$$\frac{1}{\tau_{\pm}} = \frac{1}{2\tau_{el}} + \frac{1 - \mathcal{L}}{2\tau_o} \pm \sqrt{\left(\frac{1}{\tau_{el}} - \frac{1 - \mathcal{L}}{\tau_o}\right)^2 - 4\frac{R_o\mathcal{L}(2 + \beta_I)}{\tau_o}}$$

- Where the electrical time constant is:

$$\tau_{el} = \frac{L}{R_{sh} + R_o(1 + \beta_I)}$$

- β_I describes changes in TES resistance vs current- normally 0.
- The responsivity rolls off as single poles with time-constants τ_{\pm}



Current Noise and SQUID Noise⁶

- Responsivity:

$$S(f) = -\frac{(1 - \tau_+/\tau)(1 - \tau_-/\tau)}{I_o R_o (2 + \beta_I)} \frac{1}{\sqrt{(1 + (2\pi f \tau_+)^2)(1 + (2\pi f \tau_-)^2)}}$$

- In the lab, we often approximate this to:

$$S(f) \approx -1/V_{bias}$$

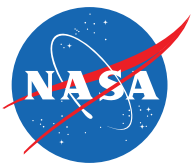
- Units: A/W. This interrelates NEI and NEP:

$$NEI_*^2 = S(f)^2 NEP_*^2$$

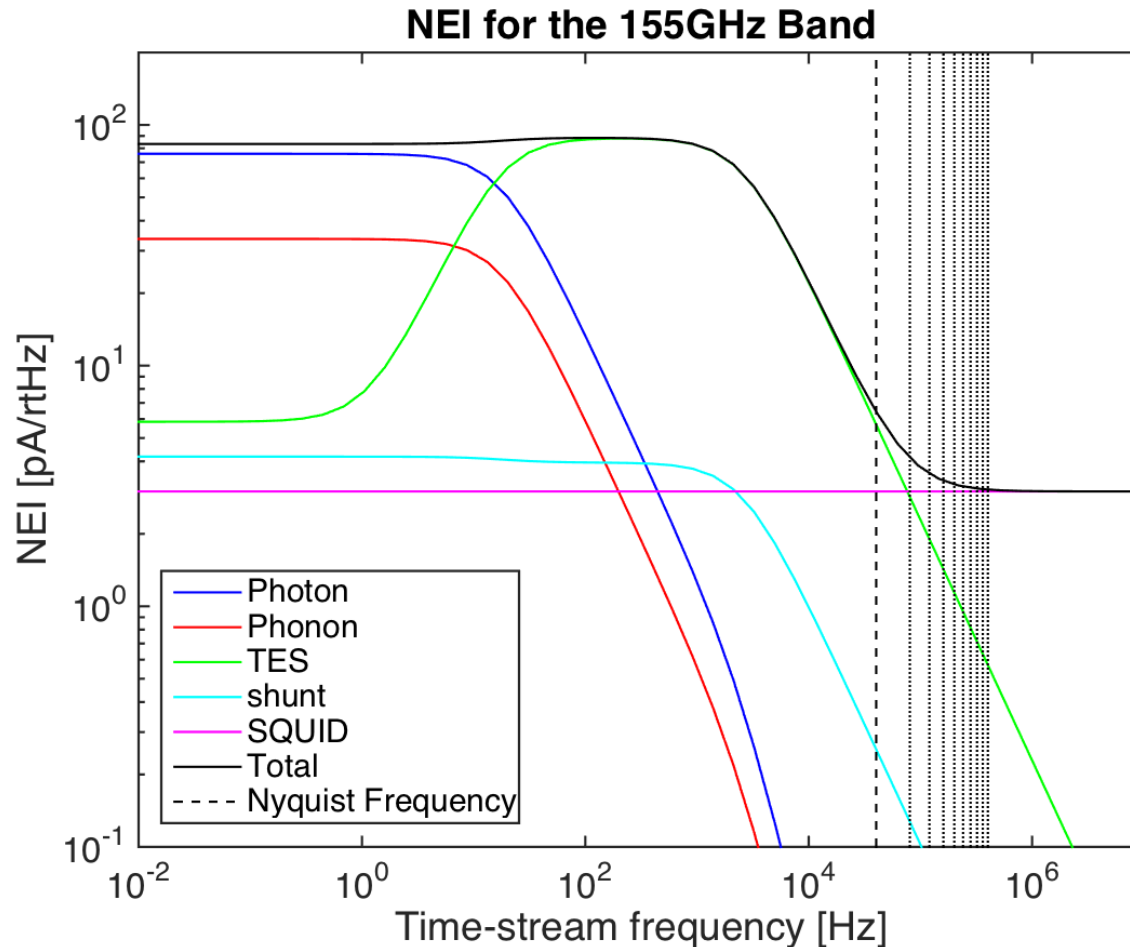
- The SQUID has white broad-band current noise. Most measured to be NEI~3pA/√Hz
- Responsivity still applies, but with a transfer function bearing an additional pole of:

$$\frac{1}{\sqrt{1 + (2\pi f L/R)^2}}$$

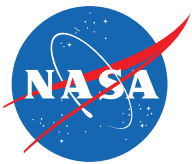
- R is the dynamic resistance of the SQUID, ~5Ω
- L is the inductance, including the Nyquist Inductor to limit aliasing, typically ~1-2uH.



NEI frequency dependence



- Curves for the 155GHz channel (Channel no 12)
- Science channel at left, probably 0.01Hz-10Hz
- No flicker noise yet
- Aliasing will fold current noise at frequency multiples of f_{Nqy} (faint dashed lines) into the in-band noise.



Aliased Noise⁷

- We roll-off high frequency electrical noise with LR circuits to mitigate aliasing. $L_{nqy} \sim 2\mu\text{H}$ in balloon and ground experiments.
- $N_{MUX}=32$ and 64 for existing systems, could increase to 128. I use 32 here.
- We must sample fast enough to detect high frequency noise set by stray inductance ($\sim 0.5\mu\text{H}$):

$$f_{Nyq} = \frac{1}{2\pi} \frac{1}{2N_{MUX} L_{stray} / R_{SQ}}$$

- but not sample so fast that detectors don't have time to settle!
- For a given audio frequency f , we can sum aliased contributions to current noise in added multiples of the Nyquist frequency.

$$NEI_{*+aliasing}^2 = NEI_*^2 + \sum_{j=1}^{j_{max}} [NEI_*^2(2j f_{Nyq} - f) + NEI_*^2(2j f_{Nyq} + f)]$$

- We terminate the sum once the sum converges
 - j_{max} only needs to be 10 for detector noise because of low frequency poles.
 - j_{max} for SQUIDs needs to be much higher (1000) because the NEI is white. Sum increases in-band SQUID noise by $\sim 50\%$.



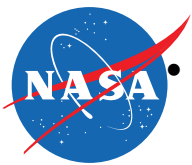
Detector White NEPs for $T_o=250\text{mK}$

Freq [GHz]	NEP_photon	NEP_phonon	NEP_TES	NEP_shunt	NEP_det_alias	NEP_SQUID	NEP_internal	NEP_total	NET_per_det [uK_CMB.rts]
20.8	3.09	3.40	0.64	0.39	0.47	1.81	3.95	5.01	81.00
25.0	3.43	3.67	0.69	0.42	0.51	1.95	4.26	5.47	74.03
30.0	3.80	3.94	0.74	0.45	0.55	2.10	4.59	5.96	67.73
36.0	4.26	4.25	0.80	0.49	0.59	2.27	4.95	6.53	61.71
43.2	4.68	4.51	0.85	0.51	0.63	2.40	5.24	7.03	56.97
51.8	5.27	4.84	0.91	0.55	0.67	2.58	5.63	7.71	51.96
62.2	5.73	5.04	0.95	0.58	0.70	2.69	5.86	8.20	48.67
74.6	6.32	5.27	0.99	0.60	0.73	2.81	6.13	8.81	45.52
89.6	6.95	5.46	1.03	0.62	0.76	2.91	6.35	9.42	43.16
107.5	7.66	5.64	1.06	0.64	0.78	3.01	6.56	10.08	41.57
129.0	8.29	5.70	1.07	0.65	0.79	3.04	6.63	10.61	41.65
154.8	8.99	5.74	1.08	0.66	0.80	3.06	6.68	11.20	43.46
185.8	9.76	5.76	1.08	0.66	0.80	3.07	6.70	11.84	48.43
222.9	10.65	5.79	1.09	0.66	0.81	3.09	6.74	12.60	59.27
267.5	11.89	5.94	1.12	0.68	0.83	3.16	6.90	13.75	81.42
321.0	13.38	6.12	1.15	0.70	0.85	3.26	7.12	15.15	135.06
385.2	15.71	6.58	1.24	0.75	0.91	3.51	7.65	17.47	265.43
462.2	18.40	7.05	1.32	0.80	0.98	3.76	8.19	20.14	663.66
554.7	21.70	7.60	1.43	0.87	1.06	4.05	8.83	23.43	2204.86
665.6	25.70	8.22	1.54	0.94	1.14	4.38	9.56	27.42	9675.04
798.7	30.17	8.82	1.66	1.01	1.23	4.70	10.26	31.87	60092.64

- Unless otherwise noted, units are aW/rHz
- $T_o=250\text{mK}$, $T_c=500\text{mK}$, $SF=2.0$
- *Not* Background limited **for lowest 8 channels**. Options: lower T_c & T_c , increase sample rate, increase Nyquist Inductor
- Single Detector NET_{CMB} at far right. NOTE: explodes above 460GHz

$$NET = NEP / (\sqrt{2} dP/dT)$$

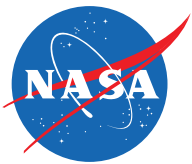
- Aliasing penalty is small: 5% at most (lowest channel) at this temp



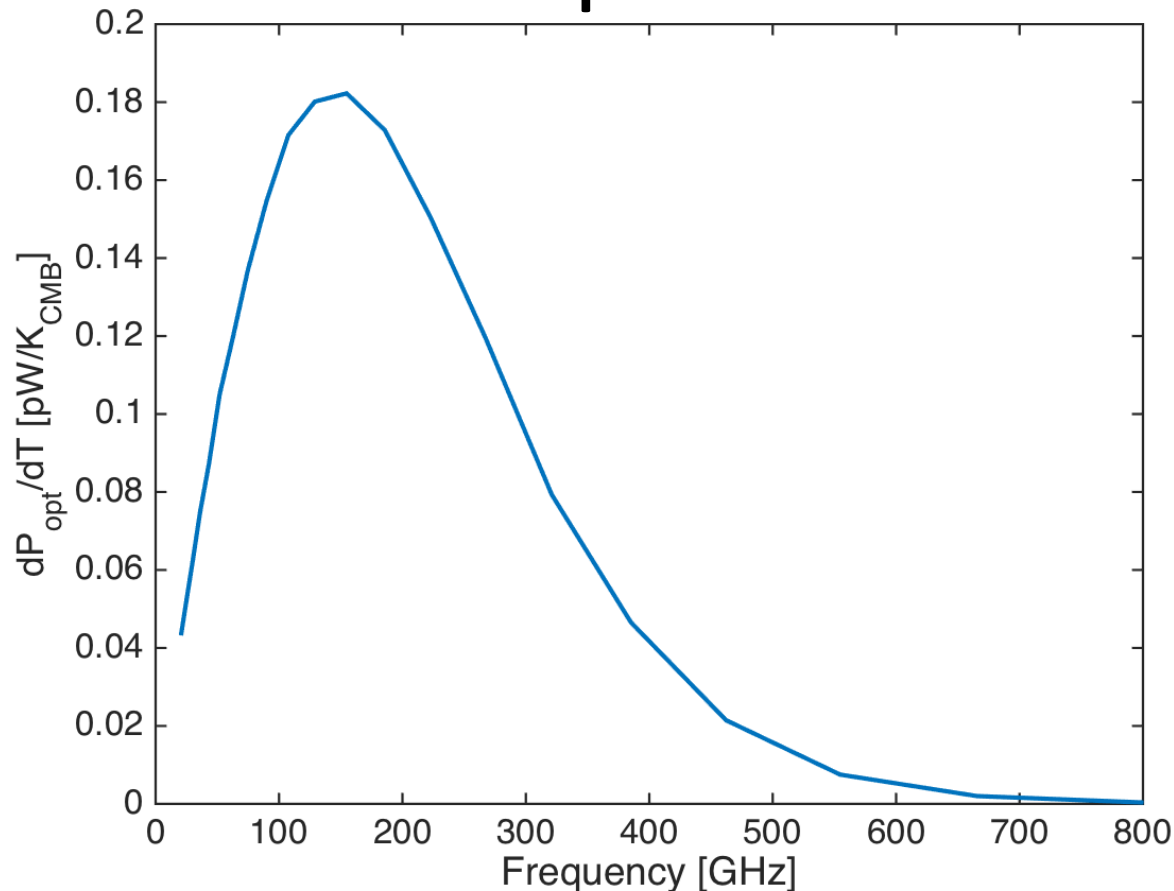
Detector White NEPs for $T_o=100\text{mK}$

Freq [GHz]	NEP_photon	NEP_phonon	NEP_TES	NEP_shunt	NEP_det_alias	NEP_SQUID	NEP_internal	NEP_total	NET_per_det [uK_CMB.rts]
20.8	3.09	2.15	0.40	0.25	0.30	1.81	2.87	4.22	68.08
25.0	3.43	2.32	0.44	0.26	0.32	1.95	3.09	4.62	62.49
30.0	3.80	2.50	0.47	0.28	0.35	2.10	3.33	5.05	57.44
36.0	4.26	2.69	0.51	0.31	0.38	2.27	3.59	5.57	52.65
43.2	4.68	2.85	0.54	0.33	0.40	2.40	3.81	6.03	48.88
51.8	5.27	3.07	0.58	0.35	0.43	2.58	4.09	6.67	44.93
62.2	5.73	3.19	0.60	0.36	0.45	2.69	4.25	7.14	42.37
74.6	6.32	3.34	0.63	0.38	0.47	2.81	4.45	7.73	39.96
89.6	6.95	3.46	0.65	0.39	0.48	2.91	4.61	8.34	38.22
107.5	7.66	3.57	0.67	0.41	0.50	3.01	4.76	9.02	37.17
129.0	8.29	3.61	0.68	0.41	0.50	3.04	4.81	9.58	37.61
154.8	8.99	3.64	0.68	0.41	0.51	3.06	4.85	10.22	39.63
185.8	9.76	3.65	0.68	0.42	0.51	3.07	4.86	10.90	44.61
222.9	10.65	3.67	0.69	0.42	0.51	3.09	4.89	11.72	55.12
267.5	11.89	3.76	0.71	0.43	0.52	3.16	5.01	12.90	76.41
321.0	13.38	3.87	0.73	0.44	0.54	3.26	5.16	14.34	127.81
385.2	15.71	4.16	0.78	0.48	0.58	3.50	5.55	16.66	253.11
462.2	18.40	4.46	0.84	0.51	0.62	3.76	5.95	19.34	637.14
554.7	21.70	4.81	0.90	0.55	0.67	4.05	6.41	22.62	2129.39
665.6	25.70	5.21	0.98	0.59	0.73	4.38	6.94	26.62	9392.52
798.7	30.17	5.58	1.05	0.64	0.78	4.70	7.44	31.08	58600.72

- $T_o=100\text{mK}$, $T_c=200\text{mK}$, same SF=2.0
- Background limited for all channels
- Aliasing is a bigger part of the picture now
- Will re-visit the lowest channels with more realistic loading model



Loss of sensitivity to CMB at high frequencies

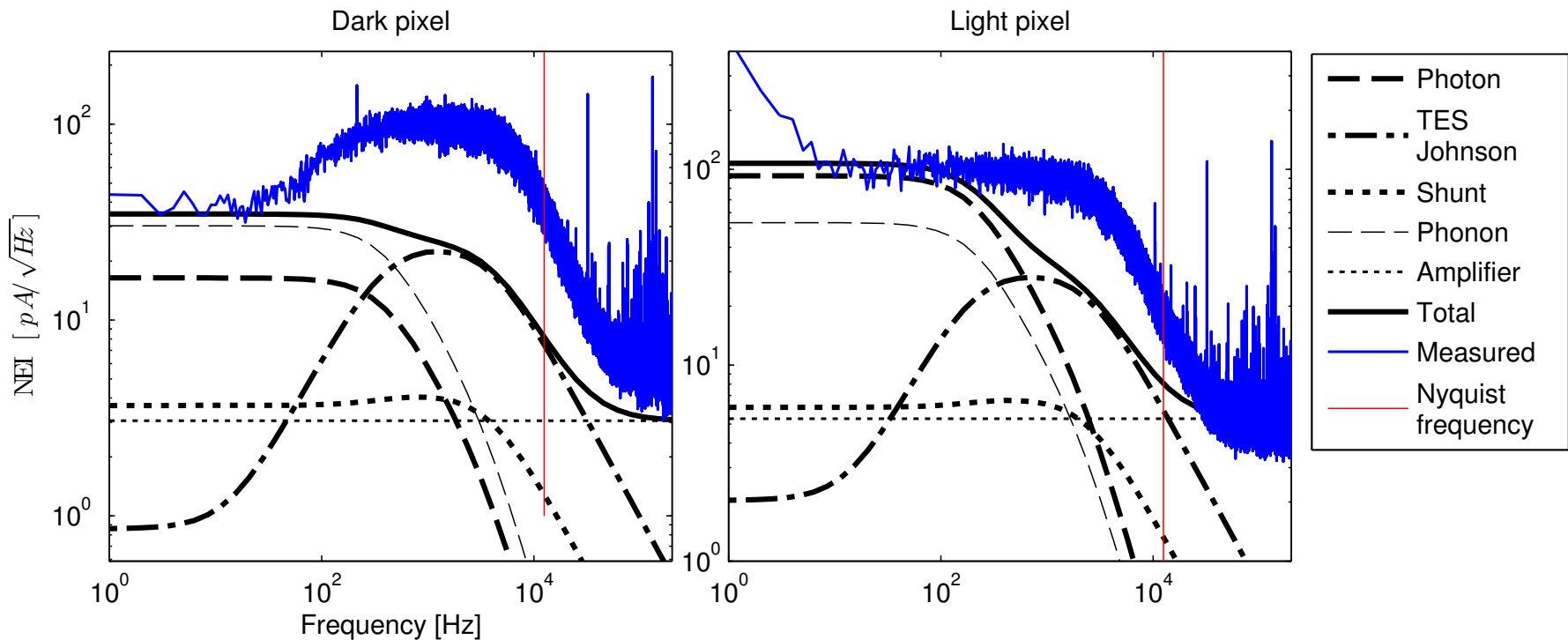


$$NET = NEP / (\sqrt{2} dP/dT)$$

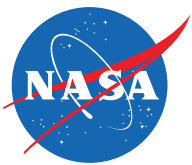
- Not really a surprise
- High channels aren't for CMB anyway
- Should be normalized to temp of galactic features, not 2.7K

Comparison with BICEP2/Keck Array¹

- Measured White Noise in Keck Array 150GHz: Dark vs light
- This is *on-the-sky*, observing from the South Pole, single detector samples



- Our model makes no attempts to model “excess noise” above the science channels
- Good Agreement between model and data for low-frequency white noise
- $1/f$ (atmospheric) ~ 2 -3Hz



Effects of aliasing

Freq_GHz	NEP_no_alias	NEP_det_alias	NEP_SQ_no_alias	NEP_SQ_alias	NEP_total	MUX_penalty
20.8	3.79	0.30	0.18	1.63	4.22	1.11
25.0	4.17	0.32	0.19	1.76	4.62	1.11
30.0	4.58	0.35	0.21	1.89	5.05	1.10
36.0	5.07	0.38	0.23	2.04	5.57	1.10
43.2	5.51	0.40	0.24	2.16	6.03	1.09
51.8	6.13	0.43	0.26	2.33	6.67	1.09
62.2	6.60	0.45	0.27	2.42	7.14	1.08
74.6	7.19	0.47	0.28	2.53	7.73	1.08
89.6	7.80	0.48	0.29	2.62	8.34	1.07
107.5	8.48	0.50	0.30	2.71	9.02	1.06
129.0	9.07	0.50	0.30	2.74	9.58	1.06
154.8	9.73	0.51	0.30	2.76	10.22	1.05
185.8	10.45	0.51	0.30	2.77	10.90	1.04
222.9	11.30	0.51	0.31	2.78	11.72	1.04
267.5	12.50	0.52	0.31	2.85	12.90	1.03
321.0	13.95	0.54	0.32	2.94	14.34	1.03
385.2	16.27	0.58	0.35	3.16	16.66	1.02
462.2	18.96	0.62	0.37	3.38	19.34	1.02
554.7	22.25	0.67	0.40	3.65	22.62	1.02
665.6	26.25	0.73	0.43	3.95	26.62	1.01
798.7	30.71	0.78	0.47	4.23	31.08	1.01

- $T_0=100\text{mK}$. MUX penalty ratios are more flattering at higher temps.
- MUX penalty runs high at the lowest channels. Might be a problem for 4K primary.
- Could increase the Nyquist inductor higher than $2\mu\text{H}$ to roll off SQUID noise at lower frequencies.

References

1. Richards, P. “Bolometers for infrared and millimeter waves,” Journal of Applied Physics Vol 76 Issue 1 (1994). Doi: 10.1063/1.357128
2. O’Brient, R. et al. “Antenna-coupled TES bolometers used in BICEP2, Keck Array, and SPIDER.” The Astrophysical Journal Vol 812 No 2. arXiv:1502.00619
3. Holmes, W. “Thermal Transport in Silicon Nitride Membranes and Far Infrared Studies of Novel Materials” UC Berkeley (1998)
4. Guildemeister, J. “Voltage-Biased Superconducting Bolometers for Infrared and mm-Waves” U.C. Berkeley (2000)
5. Rahlin, S. et al. , “Pre-flight integration and characterization of the SPIDER balloon-borne telescope.” Proc. SPIE Vol 9153 (2014) arXiv:1407.2906
6. Irwin, K., Hilton, G. “Transition Edge Sensors,” Cryogenic Particle Detection, ed. C. Enss, Spring (2005) doi: 10.1007/10933596_3
7. Niemack, M.D., “Towards Dark Energy: Design, Development and Preliminary Data from ACT.” Princeton University, 2008

